Mathematical Modeling and Simulation of Automatic Control of Wheeled Mobile Robot

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Abstract- A detailed study on the analytical study of kinematics of WMR (Wheeled Mobile Robots) is done in the following work. The objective is to develop an appropriate kinematic model of a WMR and test various time varying feedback control algorithms on this model to control its motion from a given starting position to desired goal position. Various control strategies are reviewed and compared for trajectory tracking and posture stabilization in an environment free of obstacles. From the comparison of the obtained results, guidelines are provided for WMR end-users. Three different kinematic models are developed in the following work and tested using ode23 solver of MATLAB.
The first model is a tricycle type model having two rear wheels driven independently and a front wheel on a castor. The model is tested using a time varying smooth feedback control law satisfying liapunov's criterion for stability. Various modifications in the control strategy are tested and the results are presented. The strategy is then extended so as to make the vehicle trace a number of goal positions.
The second model is similar to a conventional vehicle in which the front wheels can be steered through a range of permitted values of angle in accordance with the longitudinal speed and length of the vehicle. Different control strategies are tested on this model and modified suitably to yield satisfactory results for all situations. This strategy is also extended so as to make the model trace a number of goal positions.

Introduction

Wheeled mobile robots (WMRs) have been an active area of research and development over the past three decades. This long-term interest has been mainly fueled by the myriad of practical applications that can be uniquely addressed by mobile robots due to their ability to work in large (potentially unstructured and hazardous) domains. A nonlinear control of a wheeled robot is discussed here.

1.1 Basic motion tasks

The basic motion tasks that we consider for a WMR in an obstacle-free environment are:

***Point-to-point motion*:** The robot must reach a desired goal configuration starting from a given initial configuration.

Trajectory following **:** A reference point on the robot must follow a trajectory in the Cartesian space (i.e., a geometric path with an associated timing law) starting from a given initial configuration.

Chapter 2
Tricycle type model

The kinematic model for wheeled mobile robots is described here for the nonholonomic constraint of pure rolling and non-slipping. Based on the kinematic model, the differentiable, time-varying kinematic controllers for the regulation control problem will be analyzed here. The model will be tested in different sets of conditions for a number of control strategies and the corresponding results will be analyzed.

1.1 2.1 Kinematic model

The model is a tricycle type model having two rear wheels driven independently and a front wheel on a castor. The kinematic model for the nonholonomic constraint of pure rolling and non-slipping is given as follows.

$$q_d = S(q) * v \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

Where q(t), qd(t) are defined as,

$$q = \begin{bmatrix} x_c \\ y_c \\ \theta_c \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

$$\dot{q} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

x_c(t) and y_c(t) denote the position of the center of mass of the WMR along the X and Y Cartesian coordinate frames and θ_c(t) represents the orientation of the WMR, x_{cd}(t) and y_{cd}(t) denote the Cartesian components of the linear velocity, the matrix S(q) is defined as follows

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

And velocity vector v(t) is defined as

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ \theta_d \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

The control objective of regulation problem is to force the actual Cartesian position and orientation to a constant reference position and orientation. To quantify the regulation control objective, we define x(t), y(t), θ(t) as the difference between the actual Cartesian position and orientation and the reference position as follows

$$x(t) = x_c - x_{rc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.6)$$

$$y(t) = y_c - y_{rc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.7)$$

$$\theta(t) = \theta_c - \theta_{rc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.8)$$

x_{rc}, y_{rc}, θ_{rc} represent the constant position and orientation.

$$q_1 = x\cos\theta + y\sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.9)$$

$$q_2 = -x\sin\theta + y\cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

$$q_1 = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.11)$$

where q_1, q_2, q_3 are the auxiliary error of the system. Taking the derivatives of the above and using the kinematic model given in equation (2.2), can be rewritten as follows

$$\dot{q}_1 = v_1 + v_2 e_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.12)$$

$$\dot{q}_2 = -v_2 e_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.13)$$

$$\dot{q}_3 = v_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.14)$$

2.2 Control development:

The control objective is to design a controller for the transformed kinematic model given by equations (2.12), (2.13), (2.14) that forces the actual Cartesian position and orientation to a constant reference position and orientation. Based on this control objective, a differentiable, time-varying controller was proposed as follows:

$$v_1 = -k_1 e_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.15)$$

$$v_2 = -k_2 e_3 + e_2^2 \sin t \quad \ldots\ldots\ldots\ldots \ (2.16)$$

Where k_1 and k_2 are positive constant control gains. After substituting the equations the following closed-loop error system was obtained:

$$\dot{q}_1 = -k_1 q_1 + (-k_2 q_3 + q_2^2 \sin t)\cdot q_2 \quad \ldots\ldots\ldots \ (2.17)$$

$$\dot{q}_2 = -\left(-k_2 q_3 + q_2^2 \sin t\right)\cdot q_2 \quad \ldots\ldots\ldots\ldots \ (2.18)$$

$$\dot{q}_3 = \left(-k_2 q_3 + q_2^2 \sin t\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.19)$$

The control strategy adopted here is quiet simple. The linear velocity is directly proportional to the longitudinal error or the projected distance of the vehicle from the destination point alone. The rate at which the wheels should be turned is proportional to the angular orientation of the desired point with respect to the reference frame attached to the vehicle or the angular error and an additional time-varying term. This term plays a key role, when the vehicle gets stuck at a point. Such a situation occurs; when the longitudinal error of the vehicle vanishes that and it is oriented parallel to the desired direction, but the lateral error is not zero. In such a case, in the absence of the second term of the angular velocity control, the vehicle would get locked in that position and will fail to move any further; even though the vehicle has not reached the destination point. So this additional term is added to the steering control term. When the vehicle gets locked in the abovementioned situation, this term makes the angular error nonzero again and makes the vehicle turns a bit. This gives rise to a longitudinal error and the velocity is again non-zero. The term is sine function of time multiplied with the square of the lateral error. The sine term varies between –1 and 1 making this term vary in an oscillatory fashion. The lateral error term makes the quantity bigger when the lateral error is large. So, when the lateral error is large the disturbing steer is even larger. This quantity is smaller in comparison to the first quantity, so that the cyclic nature of the sine function does not affect the result much. Here k_1 and k_2 are positive constant control gains.

2.3 The MATLAB model:

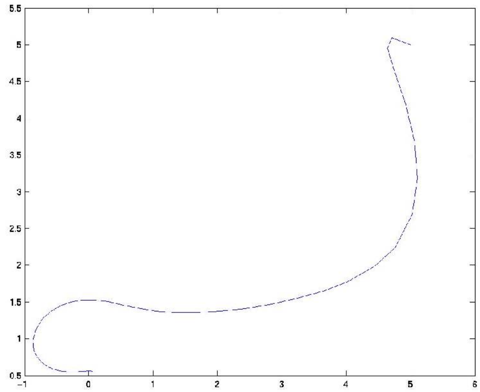
The behavior of the model and the strategy can be tested if we can obtain the trajectory of the path, when subjected to a given set of conditions. For that we need to get all the values of the state variables (q_1, q_2, q_3) at small intervals of time, which can later be plotted to obtain the trajectory of the path followed. Hence the above set of first order differential equations have to be integrated in a time interval; given the values of the initial conditions and parameters using ode23; A powerful tool of matlab. Ode23 is a function for the numerical solution of ordinary differential equations. It can solve simple differential equations or simulate complex dynamical systems. It integrates a system of ordinary differential equations using 2nd & 3rd order Runge-Kutta formulas. This particular 3rd-order method reduces to Simpson's 1/3 rule and uses the 3rd order estimate for xout. The process of ode23 is as follows: A string variable with the name of the M-file that defines the differential equations to be integrated. The function needs to compute the state derivative vector, given the current time and state vector. It must take two input arguments; scalar t (time) and column vector q (state), and return output argument qdot, a column vector of state derivatives. The above set of first order differential equations was converted into the following M-file, to execute ode23.

2.4 Testing the strategy to reach destination points:

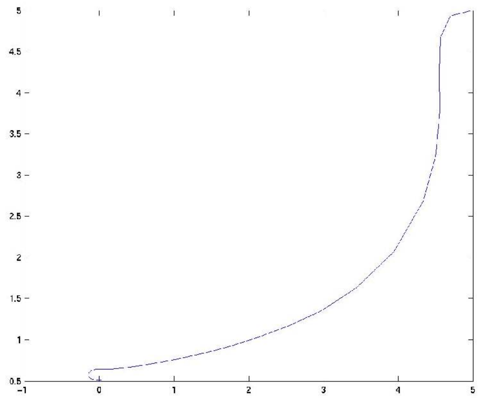
The above model is modeled in MATLAB and the strategy is tested for the vehicle to reach different destination points in all the four different quadrants. The matlab program is in Appendix I(a). Since in this model, the longitudinal and lateral errors are the state variables, the plot shows the errors decreasing and finally becoming zero. So the destination points in the plots are situated at zero and the starting points are the longitudinal and lateral errors. In the following plots, the vehicle starts with error values e_1 and e_2 as, (5, 5).

2.5 The optimal values of the parameters k_1 and k_2:

The above result shows the plots of the lateral against the longitudinal errors of the vehicle when the parameters are chosen to be $k_1 = 1$ and $k_2 = 1$. The values of k_1 and k_2 can be iterated to study their influence over the results and find out their optimal values that gives the best results. From the iteration, the best values of the parameters were found to be, $k_1 = 2$ and $k_2 = 0.1$. A comparison between the two results is shown in the following figure:



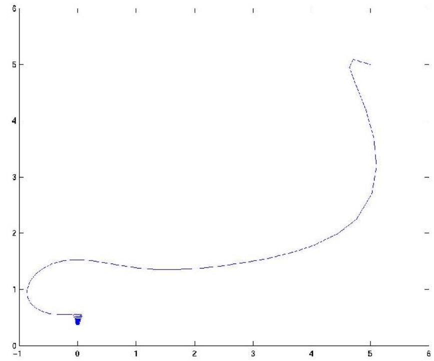
(a)



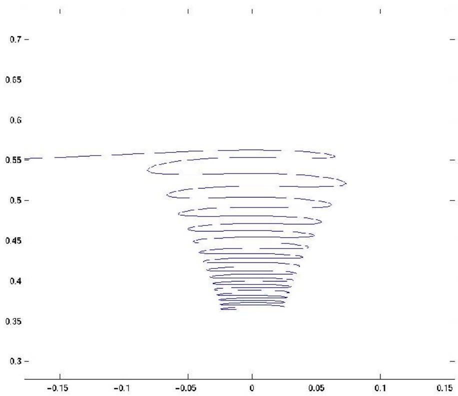
(b)

Figure 2.3: **(a) The trajectory of the vehicle when, $k_1 = 1$; $k_2 = 1$; (b) The trajectory of the vehicle with the optimal values of the parameters, $k_1 = 2$; $k_2 = 0.1$;**

Form the figures it is clear that, the lateral error (e_2) could not be completely eliminated even for the optimal values of the parameters k_1 and k_2. Still there remained a lateral error of about 0.5 units and it did not seem to improve much by the change of the values of the parameters. The amplitude of the longitudinal oscillations is however minimized; but the primary objective was to reduce the lateral error, which did not seem to be much affected by variation the parameters. The trajectory has however become reasonably smoother in the plot than the previous result. The vehicle now undergoes lesser wandering before reaching close to the goal position (see figure). The reason of this lesser wandering of the vehicle accounts for the following: By choosing a smaller value of k_2 with respect to k_1, we are assigning lower weightage to the angular error and giving more weightage to the linear errors. This directly reduces the steering rate of the vehicle. Hence the vehicle remains in more control. To understand this, let us consider driving a real car. If the driver swings the steering wheel too fast in response to curves in the road or if the steering wheel is too much free, the vehicle experiences a lot of swagger in its motion. The motion is in control or smooth if he swings the wheel slowly, keeping a firm control over the steering wheel. This exactly happens here by choosing a lower value of the parameter k_2.



(a)



(b)

Fig 2.4: **(a) the trajectory of the vehicle for 100 seconds: $k_1 = 1$, $k_2 = 1$; (b) the magnified view of the oscillatory nature of the motion.**

Plotting the trajectory for 100 seconds we find that the longitudinal error swings about 0 slowly n the amplitude decreases continuously while the lateral error approaches 0 at a very slow rate. This is clear from the figures.

The enlarged view of the final oscillatory character of the trajectory suggests that the longitudinal error swings about 0, while the lateral error very slowly nears 0. The reason for this is explained as follows. The angular velocity is sum of two components; $-k_2 e_3$ and $e_2^2 \sin t$. The presence of the sine term explains the orderly oscillations at the end part of the motion. Since the longitudinal and lateral errors become insignificant as the vehicle slowly proceeds towards the goal position, the sine term becomes the predominant factor in effecting the motion of the vehicle. This makes angular speed vary regularly with time. Since the longitudinal speed decreases as the longitudinal error decreases (the vehicle approaches close to the goal position), the vehicle travels through lesser distance in the same time in which the sine term changes sign; hence resulting in decrease of the amplitude of the oscillations as the vehicle approaches the destination point.

2.6 A simple modification in the control strategy:

So finally we tried to find out if any modification in the model could affect the results. A lot of modifications were tested and finally an introduction of a third constant k_3, yielded desirable results. When a constant k_3 was multiplied with the sine terms, the lateral error seemed to almost reduce to zero at a much faster rate. The result was smoother trajectory and faster motion of the vehicle. Higher values of constant k_3 yielded yet better results. The final oscillations seemed to almost come down to zero. The modified matlab program is in Appendix I (b).

$$v_1 = -k_1 e_1 \qquad \ldots\ldots\ldots\ldots\ldots (2.20)$$

$$v_2 = -k_2 e_3 + k_3 {e_2}^2 \sin t \qquad \ldots\ldots\ldots\ldots\ldots (2.21)$$

2.7 The results of the modification:

The modified strategy was tested for various values of state variables and parameters. The value of the parameter k_3 was iterated and it was observed that, the final result was greatly improved with the increase in the value of k_3. For value of $k_3 = 10$, the final lateral error was reduced to a very small value and also at a much faster rate (Figure 2.5). Where, $k_1 = 2$ and $k_2 = 0.1$. The vehicle starts with error values (5, 5, 1) and is simulated for 10 seconds. Figure 2.6 shows the result for $k_3 = 100$ for the same values of k_1 and k_2. It is clearly seen that the result is greatly improved, by the application of this strategy. The result is also attained at a very fast rate. With the increase in the value of k_3, the result is also further improved.

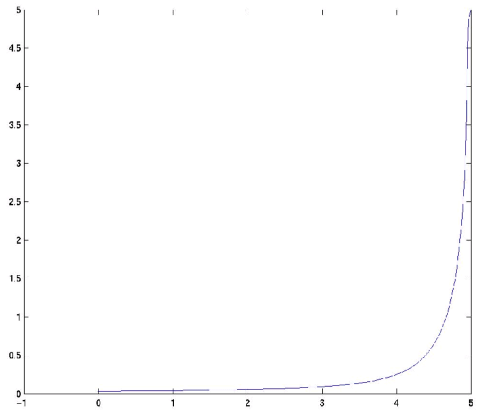


Figure 2.5: **the trajectory of the vehicle with the modified strategy: $k_3 = 10$**

In the following figure, we can clearly see the final position of the trajectory. The lateral error is 0 and the longitudinal error is less than 0.005×10^{-3}, which is small enough to neglect. So it is clearly seen how the model got modified. The lateral error is 0 in less than 3 seconds. The speed of the vehicle at this point is also small enough to assume it to be 0.

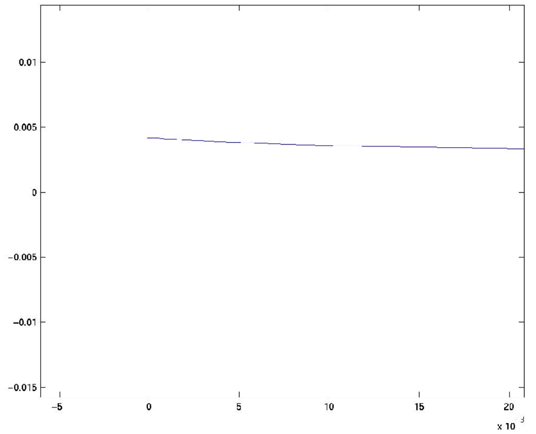


Figure 2.6: **the enlarged view**

From the above discussion, it is clearly seen that the final result has been greatly improved by the modification in the strategy. By choosing a higher value of k_3, we are actually, increasing the weightage of the second term in the angular velocity control term. That means the angle now changes faster than before, and hence the vehicle reaches to the point at a faster rate also. When the desired distance becomes very small, the angular change also becomes small. But by choosing higher values of k_3, it has become very fast and the amplitude has also increased, which makes the steps at which the vehicle nears the destination point, bigger. But since, the second term is a sine function of time; it will simply fluctuate about a mean value, with amplitude diminishing slowly.

2.8 Controls through Waypoints to Destination:

After the objective of moving to a single point was achieved satisfactorily, the next task was to extend the strategy so as to trace a number of points. Given [x_i, y_i, θ_i], i = 0, 1, 2…………..m, develop an algorithm to make the vehicle to reach x_m, y_m, θ_m, starting from x_0, y_0, θ_0, passing through the m-1 points in sequence. The above objective was attained by the following approach. A parent algorithm was made, in which all the points to be traversed were stated. The parent algorithm defines a small region of space around the way points which is known as error value. It then sets the first waypoint as the goal point for the original control strategy. When the vehicle reaches close enough to the first waypoint (in the parent algorithm), specified by the error value, it sets the next waypoint to be the destination point for the original control strategy taking its current position as the starting point. This way the vehicle travels through all the 'm-1' waypoints, till it reaches the final destination point. The control strategy used to travel between the waypoints is the same as that used to trace a single point in the previous model. The original model is however modified so that the vehicle now moves in a Cartesian coordinate plane. That means now the original coordinates of the point are taken as the state variables. Rather than taking the error values as the variables, which it did previously. The modified model is discussed in the next section.

2.9 The modified model:

The model is modeled with respect to the global reference frame. The vehicle type however remains unchanged. That is, a tricycle type model having two rear wheels driven independently and a front wheel on a castor. Given a global reference plane in which the instantaneous position and orientation of the model is given by (q(1), q(2), q(3)) with respect to the global reference system. The vehicle is to start at a position (x, y, θ) and has to reach a given point (x_d, y_d, θd) with respect to the global reference plane.

The longitudinal axis of the reference frame attached to the vehicle and the lateral axis perpendicular to the longitudinal axis. Since this reference frame's position changes continuously with respect to the global reference system, the instantaneous position of the origin of the reference frame attached to the vehicle is given by (q_1, q_2, q_3). The position of the point to be traced in the reference frame attached to the vehicle, with respect to the global Coordinate system is given by (e_1, e_2, e_3).

Where,

e_1 = The instantaneous longitudinal coordinate of the desired point to be traced with respect to the reference system of the vehicle.

e_2 = The instantaneous lateral coordinate of the desired point to be traced with respect to the reference system of the vehicle.

e_3 = The instantaneous angular coordinate of the desired point to be traced with respect to the reference system of the vehicle.

The conversions of the local values of

$$e_1 = (x_d - q_1) * \cos q_3 + (y_d - q_2) * \sin q_3 \quad \ldots (2.22)$$

$$e_2 = -(x_d - q_1) * \sin q_3 + (y_d - q_2) * \cos q_3 \quad (2.23)$$

$$e_3 = \tan^{-1}\left(\frac{y_d - q_2}{x_d - q_2}\right) - \theta \quad \ldots \ldots (2.24)$$

The kinematic model for the so-called kinematic wheel under the nonholonomic constraint of pure rolling and non-slipping is given as follows.

$$\dot{q}_1 = v_1 * \cos q_3 \quad \ldots \ldots \ldots \ldots (2.25)$$

$$\dot{q}_2 = v_1 * \sin q_3 \quad \ldots \ldots \ldots \ldots (2.26)$$

$$\dot{q}_3 = v_2 \quad \ldots \ldots \ldots \ldots (2.27)$$

v_1 = The longitudinal velocity applied to the vehicle

v_2 = The instantaneous angular deflection provided to the wheels of the vehicle

Or in other words, the angle by which the reference frame attached to the vehicle changes instantaneously. So these two variables have to be controlled by a control strategy, so that the vehicle reaches the desired point smoothly.

2.10 The Control Strategy:

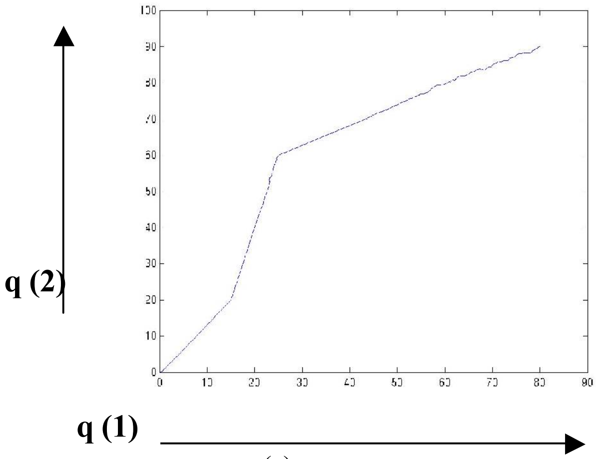
The control strategy is same as we are modifying the model only.

$$v_1 = -k_1 e_1 \quad \ldots \ldots \ldots \ldots \ldots \ldots (2.28)$$

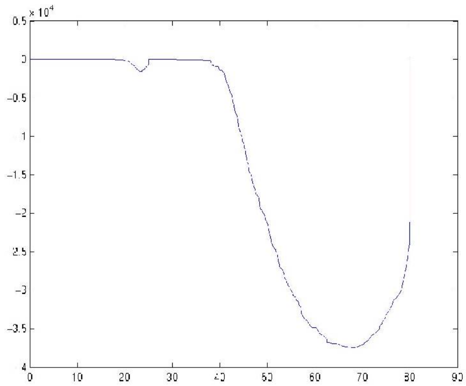
$$v_2 = -k_2 e_3 + e_2^2 \sin t \quad \ldots \ldots \ldots \ldots \ldots \ldots (2.29)$$

2.11 The results of simulation:

The above modifications are implemented in appropriate matlab models and the models are tested in ode solver. The matlab model is in Appendix I (c). Here are the results of the strategy at previous ideal values of k_1, k_2 and k_3.



(a)



(b)

Figure 2.7: **(a) Here the figure shows the path followed by the vehicle, when the above strategy is applied. The values of k_1 = 2, k_2 = 0.1, k_3 = 100; (b) The angular variation with respect to the x-distance.**

The figure 2.7(a) represents the trajectory of the vehicle, while it traverses the three points. A first look shows it to be quite all right except that the trajectory is not very smooth. But when we plot the angular variation with respect to the x coordinates, we see that there is a large angular variation (see figure 2.7(b)). The animation of the motion of the vehicle reveals that it takes a lot of turns in between the motion, which is completely undesirable. The iterative investigation associates the reason of these unnecessary rotations to the high value of the parameter k_3. Due to the high value of k_3, the weightage of the second term increases. Since the sine can have positive as well as negative values, the entire second term is positive at some times and negative at the other times. When it is positive, the whole sum of both the values becomes so high that it makes the increment in the angular velocity more than 2π at one particular instant and the vehicle takes a complete revolution. This is the reason of the large angular change in that case. The vehicle takes a lot of revolutions in midway, which is not desirable. Previously this did not happen while tracing a single point, because the vehicle completed its motion before the angular speed is raised that high to make it rotate completely, that is before the sine term reaches that high. The high variation in the angle always happened while the vehicle approached close to a waypoint, which is because the vehicle slows down as it approaches the waypoints, and thus the sine term gets time to increases to high values making angular variations high.

So by lowering k_3 and increasing k_2 we can see that the angular variations are greatly reduced. The result of k_2 = 0.25 and k_3 =10 are shown below.

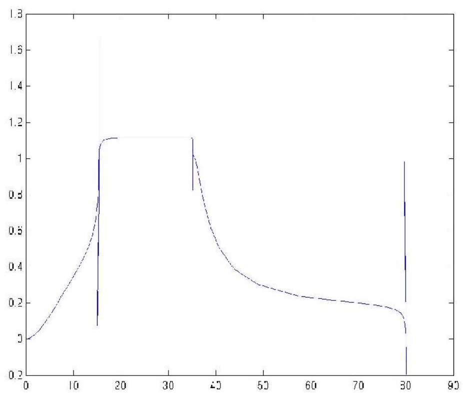


Fig 2.8: **The angular variation versus x-coordinate for the set of intermediate points (15, 5, 0.675), (35, 45, 0.927), (80, 60, 0.54). The angular variation is limited to only 1.6 radians, less than π /2**

The above result shows the effect of lowering the value of the parameter, k_3. The figures are the plots of the angular orientation q (3), with respect to the longitudinal position q (1). The angular coordinate undergoes a variation of the order of about 2 radians or one half revolutions in figure 2.8. This proves that the result has improved a lot over the previous case where the angular variation was much higher. The reason of this type of behavior is the same as before; that is the choice of high value of the parameter k_3. But the problem can not be eliminated by choosing lower values of k_3, as that would again give rise to the other problems solved by the introduction of k_3. So the problem seems to be intrinsic to the strategy itself.

Conclusion:

The strategy could trace a single point quiet successfully and the improvement in the strategy yielded very good results. As the strategy is extended for multiple points, the strategy could not perform quiet satisfactorily. The same set of parameters which yielded very beautiful results in case of a single point yielded quiet cumbersome results for multipoint tracing. It is however observed that the angular variation is reduced greatly by choosing the parameters properly. The turning of the vehicle in the middle of the motion was eliminated and the motion overall became quiet smooth, but some problems could not be eliminated. A number of approaches were tried. One of them involved the introduction of an additional parameter, k_4 with the sine term as the wave number ($e_2^2 \sin(k_4 t)$). These values even though yielded some good results, still did not quiet improve the situation. The final problems that persisted are:

- The vehicle did not always face the goal positions while moving towards them. Or in other words, sometimes the vehicle approached the goal position while facing opposite to it. This is not quiet desired in case of a real vehicle.
- The vehicle took too sharp turns (as big as 3 radians) at a single point as the vehicle approached close to the waypoints. This may be neither practically achievable nor desired in a real vehicle.
- The vehicle became quiet slow while approaching an intermediate point. It approaches the goal point at a very fast rate and becomes too slow on reaching the intermediate waypoints. Such high acceleration or deceleration rates may not be achievable in a real vehicle.
- The vehicle took a lot of time to reach to a very close neighborhood of the waypoints. So the final error values (the regions around the desired position where the vehicle has to stop finally), have to be kept reasonably higher in order to make the motion swift.

All these problems could not be eliminated completely and is the scope for future work. By choosing the parameters properly may diminish some of these problems in magnitude. However the challenge of eliminating the problems by suitable improvement in the control strategy still remains.

Chapter 3
Single track model

Introduction:

This model is similar to a conventional vehicle in which the front wheels can be steered through a range of permitted values of angle in accordance with the longitudinal speed and length of the vehicle. The case is similar to a conventional four wheeled vehicle. So the model can be described as follows.

3.1 The model:

The model is a four wheeler where the front wheels steer and the rear wheels drive. The difference of this model with respect to the previous one lies in the fact that, the previous one could have differential motion at its rear wheels whereas no differential motion could be possible in this model. Only the speed can be controlled here. It is assumed that the steering angel can be arbitrarily varied with its natural limits. The velocity is taken to be proportional to the longitudinal error, the projection of the distance of the destination. So, when the vehicle is not oriented in the direction of the destination, it has a lower velocity and it attains the maximum value when oriented in the direction of destination.

The control objective of this model is again to force the actual Cartesian position and orientation to a constant reference position and Orientation. The model is modeled with respect to the global reference frame. Given a global reference plane in which the instantaneous position and orientation of the model is given by (q(1), q(2), q(3)) with respect to the global reference system. The vehicle is to start at a position (*x, y,* θ) and has to reach a given point (*xd, yd,* θ *d*) with respect to the global reference plane.

The instantaneous position of the vehicle is considered to be at the rear end of a line segment of length l. This line segment is the line passing through the center of mass of the vehicle and parallel along the length. The longitudinal axis of the reference frame attached to the vehicle and the lateral axis perpendicular to the longitudinal axis. Since this reference frame's position changes continuously with respect to the global reference system, the instantaneous position of the origin of the reference frame attached to the vehicle is given by (q_1, q_2, q_3). The position of the point to be traced in the reference frame attached to the vehicle, with respect to the global Coordinate system is given by (e_1, e_2, e_3).

Where,

e_1 = The instantaneous longitudinal coordinate of the desired point to be traced with respect to the reference system of the vehicle.

e_2 = The instantaneous lateral coordinate of the desired point to be traced with respect to the reference system of the vehicle.

e_3 = The instantaneous angular coordinate of the desired point to be traced with respect to the reference system of the vehicle.

The conversions of the local values of

$$e_1 = (x_d - q_1) * \cos q_3 + (y_d - q_2) * \sin q_3 \quad \ldots (3.1)$$

$$e_2 = -(x_d - q_1) * \sin q_3 + (y_d - q_2) * \cos q_3 \quad (3.2)$$

$$e_3 = \tan^{-1}\left(\frac{y_d - q_2}{x_d - q_2}\right) - \theta \quad \ldots \ldots (3.3)$$

The kinematic model for the so-called kinematic wheel under the nonholonomic constraint of pure rolling and non-slipping is given as follows.

$$\dot{q}_1 = v * \cos q_3 \quad \ldots \ldots \ldots \ldots \ldots (3.4)$$

$$\dot{q}_2 = v * \sin q_3 \quad \ldots \ldots \ldots \ldots \ldots (3.5)$$

$$\dot{q}_3 = \frac{v}{\rho} = \left(\frac{v}{l}\right) * \tan\delta \quad \ldots \ldots \ldots \ldots (3.6)$$

Here, $\rho = \dfrac{l}{\tan\delta}$, which is the instantaneous radius of curvature of the trajectory of the vehicle, and

v = the longitudinal velocity applied to the vehicle

δ = The instantaneous angular deflection provided to the wheels of the vehicle

In other words, the angle by which the reference frame attached to the vehicle changes instantaneously. So these two variables have to be controlled by a control strategy, so that the vehicle reaches the desired point smoothly.

3.2 The Control Strategy:

The Control objective is to design a controller for the kinematic model given by equation (1) that forces the actual Cartesian position and orientation to a constant reference position and orientation. Based on this control objective a simple time varying controller was proposed as follows.

$$v = vpar * e_1 \quad \ldots \ldots \ldots \ldots \ldots (3.7)$$

$$\delta = cpar * e_3 \quad \ldots \ldots \ldots \ldots \ldots (3.8)$$

This simply means that, the longitudinal velocity is directly proportional to the longitudinal error in the reference system attached to the vehicle. The rate at which the wheels should be turned is proportional to the angular orientation of the desired position with respect to the reference frame attached to the vehicle. Here *vpar* and *cpar* are positive constant control gains. After substituting eq(7) and eq(8) into eq(4) and eq(5) and eq(6), the following closed loop error system was developed.

$$\dot{q}_1 = vpar * e_1 * \cos q_3 \quad \ldots \ldots \ldots \ldots (3.9)$$

$$\dot{q}_2 = vpar * e_1 * \sin q_3 \quad \ldots \ldots \ldots \ldots (3.10)$$

$$\dot{q}_3 = \left(\frac{vpar * e_1}{l}\right) * \tan(cpar * e_3) \quad \ldots \ldots (3.11)$$

3.3 The results of the simulation:

The above model and strategy was tested in MATLAB for various values of state variables and parameters. The matlab model is in Appendix II (a). Two plots are shown below in which, the vehicle starts from (0, 0, 0) and reaches a point in each quadrant. The parameters: cpar = 1 and vpar = 1. The model is simulated for 10 seconds. The plots show the trajectory of the vehicle in a plane.

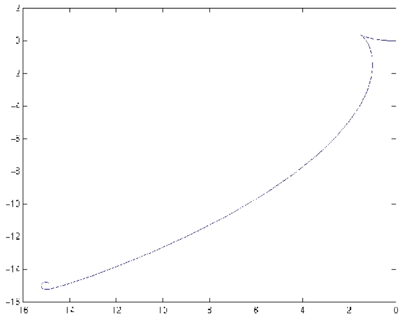


Figure 3.2**: trajectory while tracing (-5, -5)**

3.4 The problems encountered:

The model seems to perform quiet nicely from the above results except for the end part of the trajectory, where it seems to be turning about the destination point. But on more rigorous analysis, the model seems to face the following problems.

- The first and the most critical one is, the vehicle fails to start at all, when the destination point lies on the y axis; i.e., when xdes = 0. That is because; the value of ddist is zero when xdes is equal to zero. So, the velocity *v*, which is directly proportional to ddist, is also zero. Hence the vehicle does not start.
- The second problem is; after reaching sufficiently close to the destination point, the vehicle goes round about the point, without actually reaching that. This is not desired and has to be eliminated.

The above three points are discussed below and the attempts taken towards solving those problems. Here the problems are shown diagrammatically.

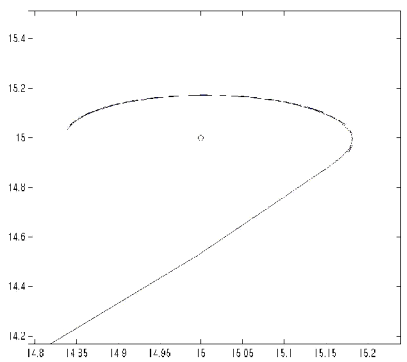


Figure 3.3: **the above figure shows, how the vehicle goes round the destination point without actually reaching that.**

In the *figure 4.6* the above-mentioned problem of the circling about the destination point is illustrated. The reason for this unwanted circling round the destination point is the following: The vehicle follows such a path that the longitudinal and lateral errors do not decrease proportionately. The longitudinal error decreases faster than the lateral error. So when the longitudinal error approaches zero, the lateral error becomes quiet large in comparison to it and simultaneously the angular error also becomes quiet large. The linear velocity of the vehicle varies proportionately with the longitudinal error, whereas the steering angular velocity is proportional to the linear velocity as well as the tangent of the angular error. Since the value of the tangent becomes quiet large when the angle approaches $\pi/2$, the angular velocity of steering also increases indefinitely even though the linear velocity is very small. That means, even though the velocity decreases considerably when the longitudinal error approaches zero, the angular velocity value increases and makes the vehicle circle about the desired point. So the problem can be solved if the linear velocity and the steering angular velocity can be controlled in such a way that the vehicle goes straight to the destination point. That means, both the longitudinal and lateral error reduce proportionately. The control may be done in the following ways.

- By choosing appropriate values for the parameters vpar and cpar.
- By choosing suitable functions for vpar and cpar and thereby controlling the linear velocity and angular velocity in the desired way.
- By making certain improvements in the control strategy to deal with the problem, if satisfactory results were not obtained by the above two strategies.

The above strategies are applied and tested in the above model. The objective of the above strategies is to smoothen the motion of the vehicle and to remove all the above-mentioned problems so that finally, the vehicle should be able to reach all the points in the plane smoothly.

3.5 The best values of the parameters:

The first strategy is employed and tested here. The objective here is to find out the best values of the parameters vpar and cpar so that the problem of unnecessary circling of the vehicle about the destination point is eliminated. So the parameters vpar and cpar are iterated, first keeping cpar constant and varying vpar to test the effect and then varying cpar keeping vpar constant. The objective of the iteration is to understand the behavior of the parameters cpar and vpar and then to find a suitable set of values for cpar and vpar, which would minimize the problem. Even though the relationship does not seem to be a direct one, still a relative understanding of how there variation affect the results, can be made. The result of iteration is discussed below. Since the problem is more pronounced, when the point to be traced is quiet close to the starting point, the destination point for the testing purpose is taken to be

(0.5, 0.5). The different results for the different values of parameters are illustrated in a table and compared with the values of parameters cpar =1, vpar=1. Some results are illustrated in graphs for clearer visualization. The hypothesis, which was made as an explanation of the unwanted results are verified with the experimental results.
From the above iterations it is clear that for the values of cpar less than 1 the steering angular velocity is too small. Hence the vehicle simply moves straight with a little amount of turning. So the vehicle covers the longitudinal distance only. By increasing the value of cpar, the vehicle performs better and more of the lateral distance is covered. For value of cpar equal to one, a drastic improvement in the lateral distance covered is observed. Then there is a great improvement in the behavior of the vehicle, when the value of cpar is chosen slightly greater than 1. Not only the lateral distance covered finally improved greatly, but also the angular position greatly reduces which indicates that the turning of the vehicle has decreased. The time taken to reach the desired point also decreases with the increase of value of vpar. That is clearly evident from the fact that, the velocity is proportional to vpar. But the drastic improvement in the time taken corresponds to the fact that the unnecessary rotations have also reduced. Since the vehicle slows down when it points away from the destination point, the less away it points, the faster it reaches the destination. As the value of cpar approaches 2, the behavior of the vehicle improves a lot. But when the value of the vpar exceeds 2, the behavior is again undesirable. For the value of vpar between 2 to 3 the behavior the vehicle resembles the behavior of cpar less than 1. So the value of cpar = 2 is the best value of cpar, with respect to vpar = 1.

Some of the above results are summarized in the figures 3.4 and 3.5 for clearer illustration. Figure 3.4 shows the different trajectories followed by the vehicle when the cpar is varied. The values that cpar assumes is 0.2, 0.4, 0.6, 0.8, and 1 respectively. The trajectories are shown collectively to show how by varying cpar the result improves as the vehicle reaches close to the goal position. Figure 3.5 displays the best trajectory for any values of vpar = 1 and cpar = 1.2, which is the best trajectory that could be found by the iteration of the values of cpar.

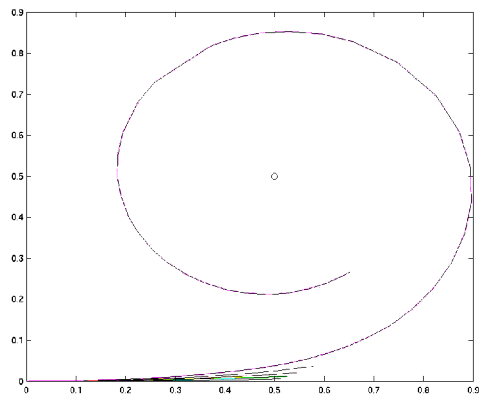


Figure 3.4: **the traces of the vehicle, when cpar varies from 0.2, 0.4. 0.6, 0.8, and 1 when, vpar = 1. The colors are respectively blue, green, red, black and magenta for the respective values of cpar.**

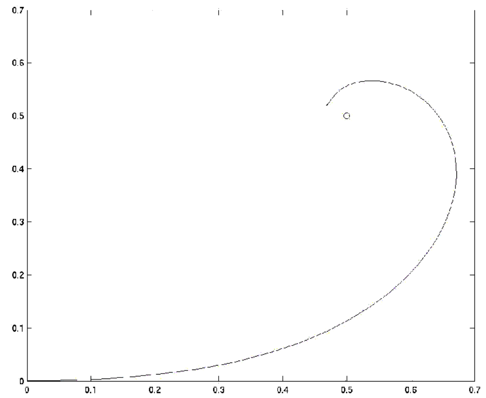


Figure 3.5: **the trace when cpar = 1.2, and vpar = 1. The point to be traced (0.5, 0.5), is marked by a circle;**

3.6 Improvements in the Control Strategy:

The problem of unwanted circling is solved to some extent by the suitable choice of the parameters. But another vital problem remains, which cannot be dealt with by the adjustment of parameters alone, but the control strategy needs modification itself to deal with the problem. That is the vehicle fails to start from its position when the longitudinal error of the destination point is zero. Or in other words the destination point lies in the line perpendicular to the starting point and passing through it. The reason for this problem lies in the control strategy itself. Since the linear velocity is proportional to the longitudinal error, when the longitudinal error becomes zero the linear velocity also becomes zero. And since the linear velocity is zero, the angular velocity automatically becomes zero, as it is proportional to the linear velocity. Hence the vehicle fails to start in that case. Suitable improvements in the control strategy may solve the problem. The following improvements are suggested to tackle the problem:

- An additional constant term can be added to the linear velocity of the vehicle, so as to make sure that the vehicle starts even when the longitudinal error is zero.
- A function can be added to the linear velocity instead of adding a constant to be more sophisticated.

The above two improvements were employed in the strategy and tested in matlab. The results are discussed below.

3.7 A Constant starting velocity:

A simple modification is made here in the parent strategy to make the vehicle trace the points that lie along the lateral axis in the vehicle frame of reference. That is by adding a small additional constant quantity to the previous equation of the linear velocity. So the linear velocity equation is modified to:

$$v = vpar * e_1 + v_0 \qquad \ldots \ldots \ldots \ldots(3.12)$$

The angular velocity remains the same as in the previous strategy. This ensures that the vehicle would start at all the conditions. But then the problem now arisen due to this strategy is, the vehicle will neither stop finally, even after reaching the point. It will continue to proceed in that direction with the same velocity v_0. This problem can be solved by choosing a strategy for the vehicle to finally stop, once it comes within a given closeness of the destination point. The strategy is as the following: the velocity of the vehicle greater than a particular error value would be according to the previous strategy and velocity of the vehicle for error less than that value is zero. The error value is the absolute distance of the instantaneous position value from the destination point. The strategy can be represented as follows:

if (abs(dist) >= near) else

$v = vpar * e_1 + v_0$; $v = 0$;

Where, dist = the absolute distance of the destination point from the instantaneous position of the vehicle.

near = the required closeness at which the vehicle should stop.

3.8 The result of simulation:

The modified strategy was developed in matlab and tested for various values of state variables and parameters. The matlab program is in Appendix II (b). Some plots are shown below in which, the vehicle starts from (0, 0, 0) and reaches a point in the lateral axis in the vehicle frame of reference in both the directions. That is, it is to reach (0, y) and (0, -y), where y is a variable. The parameters cpar and vpar are taken as, cpar = 1 and vpar = 1. The parameter v0 is iterated to find the most suitable value for it, so that the previous problem of getting stuck at certain points is solved satisfactorily. The strategy is simulated for 10 seconds. Some plots showing some typical results are also shown for clearer visualization of the solution. The following plots show the trajectory of the vehicle in a plane. The points to be traced are marked with circles.

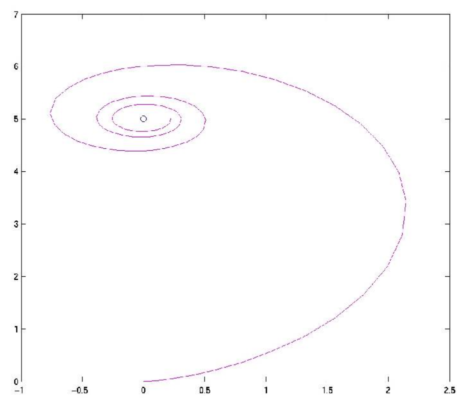


Figure 3.6: **point to be traced is (0, 5): cpar = 1, and vpar = 1.**

3.9 Controls through Waypoints to destination:

In the previous section the objective of tracing a single point was discussed with a single-track model. The various limitations of the original strategy were discussed and appropriate modifications were made in the control strategy so as to overcome those shortcomings. The next task was to extend the strategy so as to trace a number of points. Given $[x_i, y_i, \theta_i]$, i = *0,1,2…………m,* develop an algorithm to make the vehicle to reach x_m, y_m, θ_m from x_0, y_0, θ_0 passing through the *m-1* points in sequence. This is done in the following way: The vehicle starts from the starting point, (x_0, y_0) and assumes the first waypoint (x_1, y_1), to be the destination point. When the vehicle reaches close enough to the first destination point, specified by an error value (erv), it assumes that it has reached the destination point and takes the next waypoint to be the destination point. This way the vehicle travels through all the 'm-1' waypoints, till it reaches the final destination point. The control strategy used to travel between the waypoints is the same as that used to trace a single point in the previous strategy.

3.10 The result of the simulation:

The above strategy, when tested for various points, yielded very good results. Some of those results are shown below. Starting from (0, 0), the vehicle has to pass through the following points: (25, 35), (10, 60), (50, 75). The points are arbitrarily chosen and the parameters are taken to be the best values as found previously. That is, vpar = 2 and cpar = 1. The plots are shown in the Figure 3.7 (a) and (b). The motion looks quiet smooth from the trajectories that the vehicle follows.

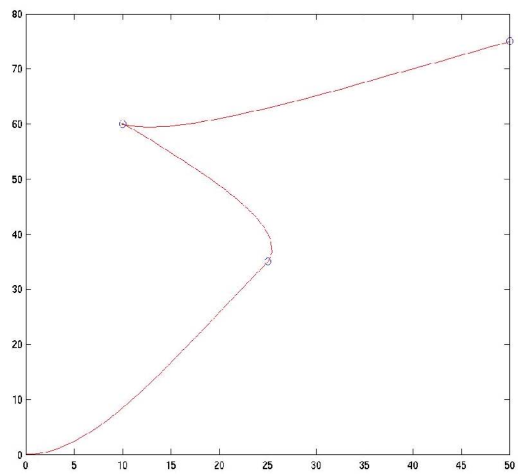


Figure 3.8: **(a) the path followed by the vehicle, while tracing (25, 35), (10, 60), (50, 75). The values of vpar = 2, cpar = 1; (b) vehicle tracing (0, 25), (40, 60), (30, 25).**

3.11 Conclusions:

The strategies still could not solve the problems satisfactorily. The problem of circling round the point, when the destination point lies close to the starting point is not solved satisfactorily and is the scope for future work. However the problem of getting locked, in case the destination point lied in the lateral axis was successfully solved. The strategy exhibited a very beautiful property. In case the destination point lied behind the starting point, instead of turning all the way round like in the previous model it traveled backwards till it faced the goal position and then moved straight to that. This is quiet similar to the way we drive our vehicles. There was not too much of swagger in the motion. However the strategy was unable to orient the vehicle in a final desired orientation. So it is not possible to park this vehicle at a chosen orientation. The strategy can be further modified to achieve this objective.

Chapter 4
Control Strategy based on three steering positions

Introduction:

In this model the problem of reaching a given point can be tackled from another approach. That can be summarized as follows. The space in the vehicle frame of reference is divided into a number of different geometric regions. The behavior of the vehicle can be modeled in a particular way according to the presence of the point in a particular region of space till it reaches in a very close vicinity of the desired point when it finally stops. Thus the behavior of the vehicle in this model is pretty predictable and hence various control strategies can be applied to the model easily. The model can be described as follows.

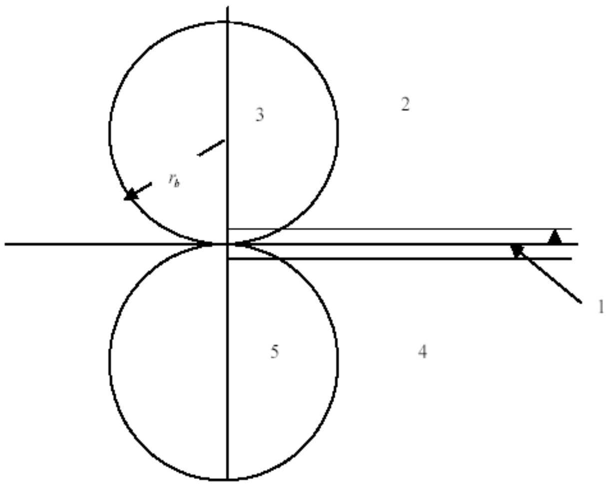


Figure 4.1: **the diagrammatic representation of the model and the different regions of space associated with it.**

The model as shown in the figure 5.1 above, is very geometric. The space is divided into five discrete regions with respect to the vehicle frame of reference. These regions can be defined as follows.

1. This portion can be defined in the vehicle Cartesian coordinate reference plane as,

   $x_l \geq 0$, and $-\varepsilon \leq y_l \leq \varepsilon$

   This is the thin rectangular strip of width 2ε along the longitudinal axis in the vehicle frame of reference.

2. This portion can be defined in the vehicle Cartesian coordinate reference plane as,

   $$y_l > \varepsilon \text{ , for } x_l \geq 0$$

   $$y_l \geq 0 \text{ , for } x_l \leq 0$$

   and, $$x_l^2 + (y_l - r_b)^2 > r_b^2$$

   This is the entire positive y plane, except for portion 1 and 3.

3. This portion can be defined in the vehicle Cartesian coordinate reference plane as,

$$x_l^{\ 2} + (y_l - r_b)^2 \le r_b^{\ 2}$$

This is the portion inside a circle of radius r_b in the positive y plane, as shown in the figure.

4. This portion can be defined in the vehicle Cartesian coordinate reference plane as,

$$y_l < \varepsilon \text{ , for } x_l \ge 0$$

$$y_l \le 0 \text{ , for } x_l \le 0$$

and, $$x_l^{\ 2} + (y_l + r_b)^2 > r_b^{\ 2}$$

This is the entire negative y plane, except for portion 1 and 3.

5. This portion can be defined in the vehicle Cartesian coordinate reference plane as,

$$x_l^{\ 2} + (y_l + r_b)^2 \le r_b^{\ 2}$$

This is the portion inside a circle of radius r_b in the positive y plane, as shown in the figure.

The instantaneous position of the vehicle is considered to be at the rear end of a line segment of length l. This line segment is the line passing through the center of mass of the vehicle and parallel along the length. The longitudinal axis of the reference frame attached to the vehicle and the lateral axis perpendicular to the longitudinal axis. Since this reference frame's position changes continuously with respect to the global reference system, the instantaneous position of the origin of the reference frame attached to the vehicle is given by (q_1, q_2, q_3). The position of the point to be traced in the reference frame attached to the vehicle, with respect to the global Coordinate system is given by (e_1, e_2, e_3).

Where,

e_1 = The instantaneous longitudinal coordinate of the desired point to be traced with respect to the reference system of the vehicle.

e_2 = The instantaneous lateral coordinates of the desired point to be traced with respect to the reference system of the vehicle.

e_3 = The instantaneous angular coordinate of the desired point to be traced with respect to the reference system of the vehicle.

The conversions of the local values of

$$e_1 = (x_d - q_1) * \cos q_3 + (y_d - q_2) * \sin q_3 \quad \ldots (4.1)$$

$$e_2 = -(x_d - q_1) * \sin q_3 + (y_d - q_2) * \cos q_3 \ldots (4.2)$$

$$e_3 = \tan^{-1}\left(\frac{y_d - q_2}{x_d - q_2}\right) - \theta \qquad \ldots \ \ldots \ \ldots (4.3)$$

The kinematic model for the so-called kinematic wheel under the nonholonomic constraint of pure rolling and non-slipping is given as follows.

$$\dot{q}_1 = v * \cos q_3 \qquad \ldots \ \ldots \ \ldots (4.4)$$

$$\dot{q}_2 = v * \sin q_3 \qquad \ldots \ \ldots \ \ldots (4.5)$$

$$\dot{q}_3 = \frac{v}{\rho} = \left(\frac{v}{l}\right) * \tan\delta \qquad \ldots \ \ldots \ \ldots (4.6)$$

Here, $\rho = \dfrac{l}{\tan\delta}$, which is the instantaneous radius of curvature of the trajectory of the vehicle, and

v = the longitudinal velocity applied to the vehicle

δ = The instantaneous angular deflection provided to the wheels of the vehicle

Or in other words, the angle by which the reference frame attached to the vehicle changes instantaneously. So these two variables have to be controlled by a control strategy, so that the vehicle reaches the desired point smoothly.

4.2 The Control Strategy:

The Control objective is to design a controller for the kinematic model given by equation (1) that forces the actual Cartesian position and orientation to a constant reference position and orientation. Based on this control objective a simple time varying controller was proposed as follows.

For controlling the velocity the following strategy is adopted.

if ($e_1^{\ 2} + e_2^{\ 2} \ge c^2$)

$v = V_0$

else

$v = 0$

end

This means that the velocity is a constant and has a value $v = V_0$, for all the points in the space except for the points inside a circle of radius c. this circle is the region in which we can choose the vehicle to finally stop. This can be chosen as small as required for the vehicle to stop at a very close vicinity of the desired point.

For the angular velocity control, the following conditional control strategy is adopted. The angular velocity fed to the system is region specific. A step angular displacement value is fed to the system as follows.

1. For region 1 the angular deviation is,

   $\delta = 0$;

2. For regions 2 and 5 the value of the angular deviation is,

   $\delta = \delta_0$;

3. For regions 3 and 4 the value of the angular deviation is,

   $\delta = -\delta_0$;

This simply means that, the step angular displacement is constant and its sign +ve, -ve or 0 is chosen according to the instantaneous location of the desired point. This simple control strategy is applied in the vehicle and tested.

4.3 The results of the simulation:

The above model and strategy was tested for various values of state variables and parameters. Four plots are shown below in which, the vehicle starts from (0, 0, 0) and reaches a point in each quadrant. The parameters are: v0 = 10, err = 0.01 and, c = 0.1. The model is simulated for 10 seconds. The plot shows the trajectory of the vehicle in a plane. The desired point is marked with a " * ".

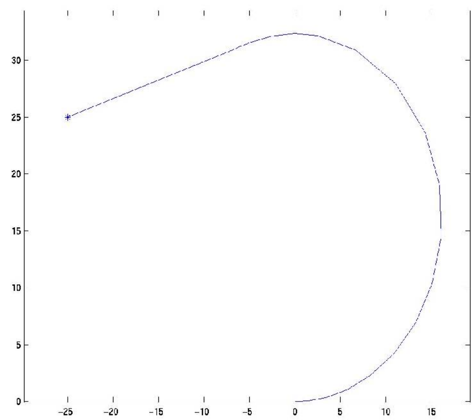


Figure 4.2: **trajectory while tracing (25, 25);**

4.4 The problems encountered:

The model seems to perform quiet nicely from the above results except for only one problem. The vehicle finally failed to stop at the destination point, when the error-range (error-range is the minimum nearness to the destination point that the vehicle is finally required to attain) was smaller than a particular value, and that value was found to be dependent upon the strip width 'ε' of the region 1. In the strategy the vehicle was required to go about a circle of radius 'r_b' till the desired point comes into region 1 in the vehicle frame of reference. Given the strategy lies in any region initially. The vehicle reaches till region 1 quiet nicely. But as soon as it reaches very close to the desired point, instead of stopping at the required error-range, the vehicle keeps on tracing circles indefinitely. The problem is shown in the following plots. In the following plot the strip width and the radius of the error-range, both are taken to be equal to 0.01.

4.5 The identification and elimination of the problem:

The problems were investigated for the possible reasons of failure. A number of predictions were made and the solutions were proposed. The first task was to isolate the region that contained the problem.

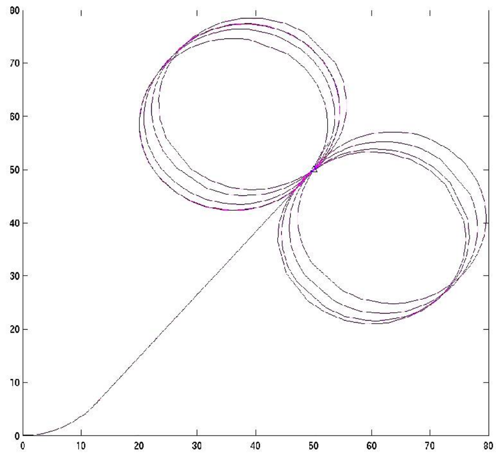


Figure 4.3: **The problem occurred when *c* was chosen to be, 0.01, which is equal to ε ; The starting point and the desired point are marked.**

From a number of iterative investigation of the above plots, it was observed that the region, which is the intersection of regions 1, 3 and 5, is the region where the problem occurs. To ensure this intuition the abovementioned region was also included inside the error range, where the vehicle has to stop finally. Thus from simple geometrical inspection, the modified error range was found to be,

$$c = \sqrt{2 \times r_b \times \varepsilon} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.7)$$

This improvement in the strategy was implemented and simulated. The result clearly showed the problems removed. So the strategy changes a bit. Now in the modified strategy, the error-range is not a parameter, but is predefined. The corresponding matlab program is in Appendix III (b).

So the results clearly indicate that the problem is centered to the above-mentioned region only. After isolating the region where the problem was centered, the next task was to investigate the real reason, why that problem occurs. Finally, the problem was found to be the following: when the vehicle touches region 1, the angular velocity instantly becomes zero. So the desired point, instead of getting into region 1, lies at the boundary and slowly proceeds longitudinally. In the vehicle frame of reference, the desired point moves along the boundary towards the vehicle slowly. This happens because of the discontinuous nature of the angular velocity shift. So finally when the desired point comes to the point where the border of the circular region intersects the region 1, it behaves according to the conditions defined for being inside region3. Hence it attains an angular velocity of either $+\delta_0$ or $-\delta_0$ and instead of stopping there, it moves away from the destination point. This way it keeps on tracing circular trajectories indefinitely, instead of reaching the desired point and stopping there. The problem can be solved, if the point lies within the region

1, instead of lying at the boundary. This can be done by another approach: By making the change of angular velocity continuous, instead of discrete.

4.6 Modifying the model to make the variation in delta continuous:

In the previous strategy, the variation in angular changes was discrete, which is not possible practically. So the model needed a modification so as to make the process continuous. That means, the angular variation will not take place in steps, but it will take place continuously. For this a fourth state variable q_4 is included in the model. This variable is the actual angular change that takes place when a step change in the desired angle change takes place. That means there is a time lag now between the required value of angular change and the actual change. This time lag factor is taken care by a parameter *k*. So now the angular change that takes place is q_4, rather than δ. The modified model is given below.

$$\dot{q}_1 = v * \cos q_3 \quad \ldots \ldots \ldots \ldots (4.8)$$

$$\dot{q}_2 = v * \sin q_3 \quad \ldots \ldots \ldots \ldots (4.9)$$

$$\dot{q}_3 = \frac{v}{\rho} = \left(\frac{v}{l}\right) * \tan q_4 \quad \ldots \ldots \ldots \ldots (4.10)$$

$$\dot{q}_4 = \left(\delta_d - q_4\right) / k \quad \ldots \ldots \ldots \ldots \ldots (4.11)$$

Here, $\rho = \dfrac{l}{\tan \delta}$, which is the instantaneous radius of curvature of the trajectory of the vehicle, and

$v =$ the longitudinal velocity applied to the vehicle

δ_d = the desired value of instantaneous angular deflection provided to the wheels of the vehicle.

This δ_d is a function of the region in which it lies. It is the same step function that was for δ. It is defined as follows.

1. For region 1 the angular deviation is,

   $\delta = 0$;
2. For regions 2 and 5 the value of the angular deviation is,

   $\delta_d = \delta_0$;
3. For regions 3 and 4 the value of the angular deviation is,

   $\delta_d = -\delta_0$;

The modified matlab model is in appendix III(c).

4.7 The result of simulation:

The result of the simulations is shown below. The vehicle starts from (0, 0), and reaches the point, (45, 55). For values of err = 0.01 and c = 0.01. The same values showed problems previously. Now the problem seems to have been solved.

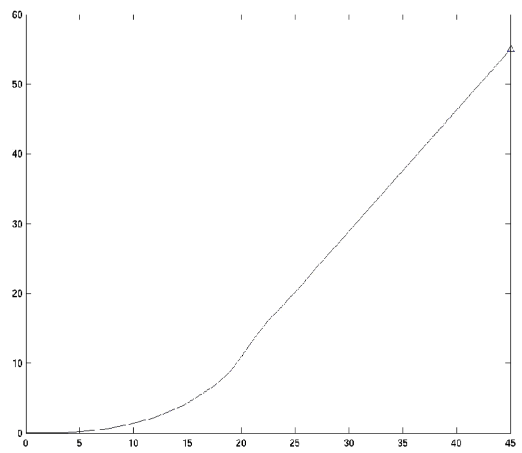


Figure 4.4: **a result showing the trajectory of the vehicle, finally successful for the desired values of parameters.**

Conclusion:

The model was a very beautiful one, and was so far the most successful in moving to a single point. However the challenge of extending the model to trace more than one point still remains to be solved. Moreover attaining a final desired orientation is still to be achieved by the use of suitable control strategies.